THE OPTION PREMIUM: UNDERSTANDING THE DETERMINANTS OF ITS VALUE

This chapter is fundamental to a professional – dare I say mature? – understanding of options. It is fundamental in that here you will appreciate an option as a *stand-alone* security. This is what I mean:

In chapters xx through yy, the focus of our discussion was the option’s payoff. We analyzed profit, loss, net P& L, of puts and calls – buying or writing, ordinary or binary – alone and in combination with the underlying and with other options. In all cases, we looked at the results of the strategies at the option’s respective expirations. At expiration, the option’s payoff, hence value, is totally dependent on the price of the underlying (and other parameters).

In this chapter, our focus is on the dynamics an option *prior* to expiration. Of course, what happens to an option prior to its expiration is crucially dependent on its payoff at expiration. But prior to expiration, an option has, in a sense, a life of its own. It is a stand-alone security. We need an example.

Consider a three-month European call on Coca-Cola, struck at 50. Coke shares are now at 45. You believe Coke will reach 48 within a week. But it will not reach 50. Further, you have no confidence that it will remain there in three months, and may well fall back to its current price by then. Should you purchase the call?

Based on our discussions in the earlier chapters, the answer is surely no. What good is an increase in the price of the underlying in a week when the option expires in three months? Furthermore, you don’t even expect it to cross the strike price, so it will never be profitable to exercise, even if exercise would be permitted!!

Ah, but here’s the thing. If Coke rises in price while the call is alive – even before expiration, and *even below the exercise price* – the call will increase in value *at that point*. And you can sell the call at a profit. Calls are bought *and sold* prior to expiration. Their prices go up and down, just like the stock itself. Of course the payoff and expiration date matter – they are important parameters in determining the call’s price dynamics. But a call (as well as a put) is a security by itself, and this chapter examines it from that perspective.

VALUE AT EXTREMES AND OTHER POINTS

Consider a three-month call option on Coke, with an exercise price of 50. Suppose the spot price of Coke shares is now 25. Would the market – i.e., investors – pay anything for this option? If Coke shares are expected to remain where they are for the next three months, obviously no. Why pay for something that is expected to produce zero income! Clearly, the option has value only if there is any expectation that Coke’s spot price changes. If it falls, the option is no worse than it is now – 0 value (the payoff cannot be negative). If it rises, and passes 50/share, there will be a positive payoff. Therefore, the option has a current value above 0 only to the extent there is any chance of it crossing 50 over the next three months. But at 25/share, Coke needs to double in price. The chances of doing slow are very slim. Hence, the call’s value is barely above 0.

What if Coke is 45/share in the spot market. The likelihood of crossing 50 is substantial. And the further above 50 it reaches, the greater the payoff. Since the value of the call reflects the possibility of crossing 50, at a spot price of 45 the premium is quite high (at least relative to the premium at a spot price of 25). In short:

*If the underlying’s spot price is below the call’s exercise price, the higher the spot price the greater the probability of crossing the strike by the expiration date. Hence, the greater the call’s value, or premium.*

If Coke is 40/share, the likelihood of it crossing 50 is still reasonable, but less than if it were at 45. The expected income of the call option is lower, hence its value is lower. Stated more formally, at an underlying spot price of 45, the *expected* payoff of the option is a function of the probability of the option ending in-the-money multiplied by the payoff at each in-the-money point. Ending in-the-money requires a price change of 5/share. At an underlying spot price of 40, the *expected* payoff of the option is also a function of the probability of the option ending in-the-money multiplied by the payoff at each in-the-money point. In this market situation, however, ending in-the-money requires a price change of 10/share, which is less likely – a lower probability. Because the expected payoff at a spot price of 45 exceeds the expected payoff at a spot price of 40, the premium is greater. Of course, at 40 the expected payoff far exceeds that of a spot price of 25, which will, therefore, be reflected in the respective premiums for these two calls.

Spot price of KO probability of crossing strike expected payoff value of call

25 extremely low minimal nearly zero

40 reasonable significantly above zero significantly above zero

45 high far above zero high

Suppose now Coca-Cola is 100/share, which is sort of symmetrically above 50 relative to 25 below 50 in the first example above. As we saw in chapter zz, at an exercise price of 50, the call option is worth at least 50, is “intrinsic value.”.[[1]](#footnote-1) If nothing changes between now and expiration, the call will earn a payoff of 50. Alternatively, the call can be sold in the market for at least 50. Assume for a moment the price of the call is 50 and consider an investor who believes Coke shares will appreciate between now and three months. She can buy Coke shares and pay 100, or buy the call and pay 50. If Coke rises to, say, 110, she can sell it for a 10/share profit. But the option will then be worth (at least) 60, and she can make the same profit of 10. On the upside, therefore, the call provides the same benefit as the underlying asset.

Coke, of course, may fall in price between now and three months. If it falls, say to 75, she stands to lose 25 with either the stock or the call. But if it falls to, say, 35, she loses 65 on the stock, but only 50 on the call (she can’t lose more than she paid for the call). In fact, the call’s loss is capped down to a price of 50 for the stock. But the stock’s loss continues down to 0. The call, therefore, is superior to the stock – they share the upside, but the call provides protection on the downside. How much protection? Protection should Coke fall below the strike price of 50. Because the call is a “preferable” asset, this investor, representing all investors – i.e., the “market,” – would be willing to pay a premium for the call compared to the stock. That is to say, she is willing to pay above the intrinsic value of the call.[[2]](#footnote-2) How much more? By the above argument, it depends on the probability of Coke falling below 50 by the call’s expiration. At a spot price of 100, that probability is quite low. Hence, the call’s value above its intrinsic value is very small.

On the other hand, if Coke is now 55/share, so that the call’s intrinsic value is 5, it is vastly superior to the stock. Again, both share equally in the upside potential of Coca-Cola stocks. Whereas a stock purchase presents the potential of losing the entire 55, the call’s loss is bounded from below by the strike price. The probability of crossing 50 given a spot price of 55 is quite substantial. Therefore, the protection the call provides is very high. Investors will pay a hefty premium above the intrinsic value for this protection.

*If the underlying’s spot price is above the call’s exercise price, the lower the spot price the greater the probability of crossing the strike by the expiration date. Hence, the greater the call’s value, or premium, above its intrinsic value.*

At a spot price of 60, the probability of Coke crossing 50 is not as high as it is at 55. The call still provides protection that the stock does not. But the protection is not as valuable. Therefore, the call’s value *above its intrinsic value* of 10 is not as great as it is at a spot price of 55.

Spot price of KO probability of crossing strike value of protection value of call

55 high well above zero well above intrinsic value

60 reasonable significantly above zero somewhat above intrinsic value

100 extremely low minimal nearly zero above intrinsic value

What if Coke is 50/share – the option is at-the-money? Let’s continue with the above reasoning. The call has no intrinsic value; i.e., if Coca Cola shares remain where they are, the option produces no income at expiration. But the chances of crossing 50 are very high. In fact, at 50, the chances of crossing 50 are higher than at any other point! Compared to any point below 50, the expected payoff is highest.

What about compared to a point above 50? Remember, when the call is in-the-money, the *extra* benefit of the call versus owning the underlying reflects the protection it provides should the underlying decline below the exercise price. But it’s already *at* the exercise price. Hence, there can be no greater protection. We conclude again that the value of the call above its intrinsic value – which is zero at this point – is greatest when it is at-the-money.

THE OPTION CURVE

Figure 1 plots the call’s value – its premium – against the underlying asset, Coke shares, using the points in the two Tables above plus the ATM point just discussed. Then it “connects” the points to create a smooth graph.

This is a crucial graph. Note that the call’s intrinsic value as a function of the underlying price is displayed in the figure as well. It consists of two straight lines, in a sense spliced together. By contrast, the call premium is a curve. At very low prices of Coke shares, the call is worth next to nothing. We might refer to this area as “far out-of-the-money.” Importantly, the curve has a slope nearly equal to 0. At very high prices of Coke shares, the call is worth a lot. But nearly its entire value is due to its intrinsic value. Its slope, therefore, is essentially 1. We can refer to this area as “deep in-the-money.” Between these extreme points, the relationship between the value of the call option and the price of the underlying displays a healthy curvature. This curvature is the essence of the option’s character. As the share price of Coke increases from a very low point – say 20 – and leaves the far out-of-the-money area, the call begins to have value. As Coke increases further – say in the 20 to 40 range – so does the call, but not by as much. Why not? The curve’s slope is less than 1.

Once Coke crosses 40, the graph shows a clear acceleration of the call’s price. The slope of the curve is still less than 1 – indeed, it never quite reaches 1 - but, crucially, it increases.[[3]](#footnote-3) For example, at a spot price of 20, the slope may be .25; a 1 point increase in price of Coke shares elicits a quarter point increase in the call’s price. At 40 the slope is .35, so that the same 1 point increase in the underlying stock price causes an increase of .35 in the price of the call. From that point on, the call really gathers steam. At a price of approximately 50, the slope reaches .5.[[4]](#footnote-4) Another way to say this is that when the call is close to at-the-money, approximately fifty percent of the stock’s price movement is mimicked in the call’s price. As the stock price rises further, the call’s premium increases as well, with the slope getting closer to 1. Every successive increase in the stock price brings forth an increase in the call’s premium and an increase in the slope. But the slope can never reach 1. Why? Because there is always *some*, however minute, protection provided by the call that is paid for by the premium being in excess of the intrinsic value, though that excess approaches, but never reaches, zero.[[5]](#footnote-5) Still, in the area where the slope is nearly 1, approximately 100% of the stock price movement is reflected in that of the call. In this region, therefore, the call and the stock are almost perfect substitutes.

Let’s go in the other direction, starting from very high prices of the underlying. When Coke is at 100, the call premium’s slope is essentially 1. A decline in the spot price is matched, approximately, by a drop in the call’s price. As Coke shares decline further, the call’s downward path decelerates. Why” Because the slope of the curve begins to decline – every point drop in the underlying’s price is accompanied by a *smaller and smaller* drop in the option’s price. Unlike the stock itself, the call presents a “cushion,” which is just another representation of the “protection” element. In the neighborhood of 50, the slope hits .5 – only half the stock’s decline is mirrored in the option. Further declines in the share price push the slope nearer to 0, the region where declines in Coke have essentially no effect on the call (as it has already reached 0).

REVIEW QUESTIONS

Raise the exercise price from 50, as analyzed in the text, to 60.

1. If Coke is 25/share, why would an investor pay anything for the call?
2. If Coke is 45/share, why is the call worth more compared to a)
3. If Coke is 100/share, why would an investor pay more than the intrinsic value for the call?
4. If Coke is 65/share, why is the call’s excess value above its intrinsic value greater compared to c)?
5. We know from chapter zz that a call with a higher strike price must have a lower value than one with a lower strike, all else the same. Consider the call on Coke struck at 50, as analyzed in the text, and the call in this example, struck at 60. Assume Coke shares are at 45. In the context of your answer to b), why is the 50-call worth more than the 60-call?
6. Assume Coke shares are at 65. In the context of your answer to d), why is the 60-call’s call’s excess value above its intrinsic value greater than that of the 50-call?
7. Harder: We know that a call with a higher strike price must have a lower value than one with a lower strike, all else the same. How can you reconcile this with the answer to f)?

PROTECTION AND PARTICIPATATION

The previous section may be summarized in the following way: As the price of the undying asset rises from a point far below the strike price, the call option prices rises, though very weakly. As the underlying increases further, the option price does as well – not to the same degree as the stock, yet *at an accelerating pace*. The acceleration continues, yet the call’s increase will never quite equal that of the stock.

Conversely, when the underlying asset price declines from a very high level relative to the strike, the call’s price falls along with it – nearly, but not quite, one-to-one. As it falls further, the call does, too, but *at a decelerating pace*. The deceleration continues, until further drops in the stock price elicit hardly any response in the call at all.

Compare now the two graphs in the figure: the option’s premium – its price, or *total* value – and its intrinsic value. Intrinsic value was the focus of chapter III, and much of chapters xx & yy revolved around it. The intrinsic value curve is flat (slope = 0) until Coke’s spot price hits 50, the strike. In other words, on the expiration date, there is no gain – no participation – to the call buyer from any increase in Coke as long as it does not cross 50. Not so for the premium. As long as the curve has a positive slope – which is essentially anywhere except the far out-of-the money region – any upward movement in Coke’s price *prior to expiration* results in an increase in the price of the call.

This is a crucial result, and one of the most important concepts in this book. And it has powerful implications. Here’s one: Suppose Coke is now 45 per share and you expect it to increase to 48, but no further. Is buying a three-month call option on Coke, with an exercise price of 50, a proper strategy? It depends. If your focus is solely on the expiration date, then of course not – based on your expectations, the option will expire worthless. But if your view is that the stock’s price appreciation will occur, say, within a week, the option’s curve tells us that the call will increase in value, allowing you to sell it at a higher price!

Another way to say this is that the call option provides a degree of participation in the stock’s price upward movement even when the option is out-of-the-money! Recall that on the expiration date, participation occurs only once the stock passes the strike price; i.e., when it is in-the-money. The other side of this is that past the strike price, further appreciation of the underlying increases the call’s payoff 1-for-1 on the expiration date. Prior to expiration, the participation rate is less than 100% (slope < 1).

On the other hand, precisely because the slope is less than 1 does the call provide a degree of protection even if it is in-the-money. Recall again that on the expiration date, should the stock be above the strike, a decline in its price produces a 1-1 drop in the call’s payoff. Prior to expiration, as the curve shows, the call premium decreases less than 100% of the stock price decline. The other side of this is past the strike price, further depreciation of the underlying causes no further loss on the expiration date. Prior to expiration, however, the protection is incomplete – the premium decreases even when it is out-of-the money (slope > 0).

TIME VALUE

The area between the option curve and the pair of straight lines in Figure I is the excess of the call’s premium above its intrinsic value. We showed above that this excess reflects the probability, and the gain associated with, crossing the strike price – from either direction – before the option expires. The longer the time to expiration, the greater that probability. This excess, therefore, is known as the call’s *time value*. In equation form.

Call Premium = Intrinsic Value + Time Value

Time value is greatest in the neighborhood near the option’s exercise price. This neighborhood is sometimes termed “near-the-money.” As the stock price moves further from the strike in either direction, time value decreases. As the stock price declines below the strike, the total premium value decreases because time value falls and intrinsic value is zero. As the stock price increases above the strike, intrinsic value rises by more than time value declines.

Table III illustrates these concepts. The examples, are the various situations explored in Tables I and II. When Coke is 25/share, the call’s premium is, say, 5 cents. It is way out-of-the-money, so it has no intrinsic value. Its time value is barely above 0, reflecting the improbability of Coke crossing 50/share by expiration.

Coke Spot Price Intrinsic Value Time Value Total Premium

25 0 .05 0.05

40 0 .75 0.75

45 0 1.00 1.00

50 0 2.50 2.50

55 5 1.00 6.00

60 10 .75 10.75

100 50 .05 50.05

The concept of an option’s time value carries enormous implications, two of which are the following:

1. All else the same, a call option with a later expiration has more time value than an otherwise similar option with an earlier expiration. For example, a six-month call on Coke exercisable at 50 has more time value than the 3-month call examined above. Because they have equal intrinsic value (due to their common strike), then by the above equation, the 6-month call’s premium exceeds that of the 3-month call.

Figure II shows the 6-month curve above the 3-month for all prices of the underlying. The two curves becomes indistinguishable at very low prices of the underlying and at very high prices. This must be so – the above analysis showed that when the call is deep-in or far-out –of-the money, time value approaches zero.

The distance between the curves is greatest near-the-money. This is eminently sensible. Time value reflects the likelihood of moving crossing the strike price. There is no point that presents a higher likelihood of moving away than that point itself. Hence, just as the 3-month call displays its greatest time value when It is near-the-money, so does the excess value of the 6-month call above that of the 3-month reach its highest point in that same neighborhood.

1. Let’s examine a call’s value *over time*. Assume nothing changes but the calendar. That is, assume the spot price of Coke remains where it is, as does investors’ assessment of Coke’s volatility. After one month, the six-month call has five months remaining. It has become, for all intents and purposes, a five-month option. We concluded above that a shorter expiration means a smaller premium. This proves that an option loses value over time. Graphically, the curve moves in a southeasterly direction. Indeed, another two months later the call becomes the three month call in the Figure. Proceeding in this manner, it is clear that as the option approaches its expiration date, the curve collapses to its intrinsic value. This dynamic is known as the option’s “time decay.” Note that the degree of time decay is greatest in the near-the-money range (because that is where the *value* of time is greatest).[[6]](#footnote-6)

Before we go on to analyzing puts, it is worthwhile pointing out that the true “option” portion of an option’s value is its time value. Intrinsic value is not unique to options. In the absence of interest rates, a forward contract between a buyer and seller to transact 100,000 shares of Coca Cola, whose spot price is 55/share, in three months at a price of 50 is obviously worth (55−50)x100,000 = $500,000 today.[[7]](#footnote-7) The buyer must pay this amount to the seller today. This has nothing to do with optionality. Rather, it is inherent in the contract – it is the contract’s *intrinsic* value. To this amount the call options adds the *choice* of exercise to the buyer’s position in the forward contract. This is its time value.

PUTS

Analyzing the put’s value and its price curve follows similarly to calls. We can be relatively brief. Consider a one-year European-style put option on Coke, struck at 50. This is the same option analyzed above, but a put instead of a call. Suppose Coke shares are trading in the cash market at a price of 100. If Coke remains where it is – more correctly, it is 100 at expiration – the put expires worthless. The put has value only to the extent there is a chance of Coke falling below 50 before expiration. As the probability is close to 0, the option essentially has no value. At a spot price of 60, the probability of passing 50 is meaningful, giving the put some value. At 55, the probability increases, hence does the premium. We conclude that for an out-of-the-money put, the lower the price of the underlying, the higher the premium.

Suppose Coke’s cash price is 25. The put’s intrinsic value is 100-25=75. If nothing changes, the put will pay off 75/share at expiration. The questions is: is it worth more than 25? Compare it to a short position in Coke. Should Coke’s price drop to 20, the short earns 5 and the put’s payoff increase by 5. If Coke rises, the short loses money, and the put payoff decreases by the same amount. But Coke’s price can go through the roof, and the short is completely naked. The put owner, on the other hand, can only lose the payoff – his losses are capped. How much is this cap worth? Not much, because it only comes into existence were Coke to jump through 100 from 25 by the one-year expiration date, a minimal probability event. The put’s value above its intrinsic value is essentially zero. At a spot price of 40, Coke has a reasonable chance of piercing 50 in a year’s time. The limit to the potential losses that the put provides compared to the short position is significant. Hence, the extra value above its intrinsic value is significant as well. At 55, the put’s superiority is great. On the way down, its payoff increases in size equal to that of the short. On the way up, its protection kicks in after only a five-point move in Coke, compared to the total nakedness of the short position. A trader would pay a lot – above intrinsic value – for this tradeoff.

Finally, at a Coke price of 50, the put’s optionality is greatest. For any downward movement in Coke, the option pays off immediately. And for an upward movement, there is no loss at all.[[8]](#footnote-8)

REVIEW QUESTIONS

Fill in a Table for Puts similar to that above for Calls:

Spot price of KO probability of crossing strike expected payoff value of put

25

40

45

50

55

60

100

THE PUT PREMIUM CURVE

Figure 3 plots the Put premium against Coke share price, using the points analyzed above and filling in by “connecting the dots.” The option’s intrinsic value is shown as well. Opposite to the Call premium curve of Figure 1, the Put is nearly worthless at very high prices for the underlying. It is “far out-of-the-money” and the slope is essentially zero. At very low prices for Coke, the put is worth a great deal. But almost all its value there is intrinsic, not due to any optionality; i.e., it is “deep in-the-money.” Its slope is *minus* 1.[[9]](#footnote-9)

Between these extremes, two characteristics stand out:

* The relationship between the Put and the underlying is a curve.
* The slope of the curve is negative throughout.

Beginning, say, at 100, as Coke falls in price, the put premium increases. As explained above, this reflects the fact that as Coke declines toward the strike, the probability of crossing it prior to expiration rises. Furthermore, the premium’s rate of appreciation increases – the slope rises in absolute value.[[10]](#footnote-10) After Coke crosses the strike and continues to decline, the probability of rising back above 50 recedes. However, the Put is now in-the-money. Every dollar decline in Coke adds a dollar to intrinsic value, more than offsetting the loss in value due to the lower probability of jumping back above 50. In short, the premium continues to increase. In fact, it continues to accelerate – the slope approaches −1.

Analyzing the dynamics in the other direction, the curve’s slope is essentially 1 in absolute value when Coke is very low. An increase in Coke’s spot price reduces the put’s premium by the (nearly) same amount. This is, of course, true of a short position in Coke – any increase in price produces a loss of the same amount. As Coke continues to rise, the Put falls in value, but less than 1-for-1. A short stock position, on the other hand, adds to its loss 1-for-1. The deceleration continues until an increase in Coke has (almost) no negative effect on the Put – the slope becomes 0. This is the cushion a Put provides that a short position does not – the essence of the option, and why there is a premium.

PUT PARTICIPATION AND PROTECTION

The discussion of the above section can be phrased in terms of protection and participation. As with the call, this is brought into sharp relief by contrasting the premium curve in Figure 2 with the intrinsic value shown there. From a point far out-of-the-money, say 100, a decline in price of Coke shares elicits an increase, albeit small, in the Put premium. The intrinsic value curve exhibit no such reaction. As we saw in chapters xx and yy, the put’s payoff – as manifested in the intrinsic value curve – remains at 0 until the underlying crosses the strike. It provides no participation in a stock price decline at all. Prior to expiration, on the other hand, as shown by the premium curve, the put does provide participation. it’s value increases – indeed, at an accelerating rate – even when it is out-of-the-money. The practical implication is clear: Suppose you expect Coke to decline in price from its current value of 45 to 40 within a week. Does a three-month Put struck at 35 do you any good? Sure it does! Suppose you pay a premium of 1 for the Put. Should Coke decline to 40, the put – despite being out-of-the-money – increases in value, say to 2.[[11]](#footnote-11) Had this occurred on the expiration date, your entire premium would be lost.

On the other hand, as the graphs show, once Coke falls in price below the strike, the intrinsic value increases one-for-one, whereas the put premium does not. Sure, it keeps getting closer to 1-for-1, but it never reaches it. But the benefit of the Put in this in-the-money region lies in the obverse of that observation. An increase in Coke’s price causes a one-for-one reduction in the payoff. The premium falls by less than 1-for-1. Thus, the put provides some measure of protection, even when it is in-the-money. Once Coke rises above 50, further increases in price do not affect the payoff. But the put premium continues to fall, though deceleratingly. The protection is incomplete.

TIME VALUE

The excess of the put’s premium above its intrinsic value – the area between the option curve and the straight lines in Figure 3 – is the put’s time value. As explained above, this excess reflects the probability, and the gain associated with, crossing the strike price from either direction before the put expires. The longer the time to expiration, the greater that probability. Therefore,

Put Premium = Intrinsic Value + Time Value

The put’s time value is greatest when it is near-the-money. As the stock price moves away from the strike in either direction, time value declines. As Coke rises above 50, the total premium decreases, because time value falls and intrinsic value is zero. As the stock price drops below 50, intrinsic value rises by more than time value declines. Table IV, the counterpart to Table III for calls, illustrates how intrinsic, time and total values react to movements in the price of Coke, given an exercise price of 50.[[12]](#footnote-12)

Coke Spot Price Intrinsic Value Time Value Total Premium

25 25 .05 25.05

40 10 .75 10.75

45 5 1.00 6.00

50 0 2.50 2.50

55 0 1.00 1.00

60 0 .75 0.75

100 0 .05 0.05

Figure 4 includes a six-month put premium curve along with the three-month just analyzed. Both are stuck at 50. The six-month option has more time value than the three-month. At any price of Coke, the six month has a greater chance of crossing 50 from either direction. The extra time value is greatest at-the-money, and recedes from there in each direction. At extremes, the two curves are indistinguishable.

Over time, with all else unchanged, each put’s time value declines. In particular, after three months, the six-month put looks like the three-month put does today. Three months later, when the put expires, the “curve” becomes the intrinsic value lines. This time decay process is sharpest near-the-money, and almost non-existent deep-in and far-out-of-the-money.

GREEKS

DELTA

In finance, Greek letters are used to denote relationships between variables. In options, they refer to the reaction of an option’s premium to changes in market variables and options contract parameters.

The *delta* of an option measures the degree of reaction of the premium to a change in the underlying’s price. Looking at Figures 1 and 3, you can see that the delta is simply the slope of the curve. Consider first the call. The delta when it is far out-of-the-money is close to 0. As Coke increases in price, eventually it brings the call up along with it – the delta increases. When Coke is 45 for example, the delta seems to be around 0.4. This means that a change in the price of Coke shares – *in either direction* – causes the call premium to move by approximately 0.4 times as much. For example, say the premium equals 1. An increase in the spot price of Coke to 45¼ results in the call increasing by .4×.25=.1 to 1.1. A decrease in the spot price results in the call decreasing by .1 to 0.9.[[13]](#footnote-13)

When Coke’s price increases to the point where the call is near-the-money, the delta is around .5. In this region approximately 50% of the spot price’s movements is mirrored by the call premium. As Coke continues its ascent, the call premium rises as well. The delta is increasing, too, which implies that the premium is accelerating. When the option is deep in-the-money, the slope is nearly 1, so that call premium changes nearly equal spot price changes. On the way down, the premium’s decline nearly duplicates that of the underlying stock. But then it begins to slow down – it decelerates. Every successive decline in the spot price is associated with less and less of a decline in the option price. Until it reaches almost zero.

APPLICATIONS OF DELTA

Talk to an options professional and she might say, “options are all about the delta.” Well, not quite (not the least of which is that there are quite a few more Greeks), but you see its importance. Here are a few applications. We’ll come across more and the book progresses.

Risk Exposure

Assume your portfolio contains 100,000 shares of Coca-Cola, currently priced at 45/share. The value of your Coke shares is 100,000×$45=$4,500,000. Suppose you are worried about a 10% decline in the price of Coke stock over the next three months. You might have arrived at this number by calculating the standard deviation of Coke share price movements over three months from historical data.[[14]](#footnote-14) A 10% decline in price translates into a loss of

.1×$4,500,000=$450,000

But what if instead of cash shares you own 100,000 calls on Coke, expiring in three months and with an exercise price of 50? You haven’t really “invested” in Coke. Say the premium is $1/share. By the definition of delta, the call’s price responds to that of the underlying according to:

∆Call = delta×∆Stock

The call’s delta is 0.4. A 10% decline in Coke stock price – from 45 to 40.5 per share – results in a loss of

.4×4.5×100,000 = $180,000

The risk exposure of your options position is 40% that of the stock position. Note that your risk is not directly dependent on the option’s premium – it does not enter the above equation! Rather, the crucial ingredient is the delta.

What we have just examined is the risk of the call option arising from movements in the price of the underlying. This is not the only risk factor. But it is the one captured by the call’s delta.

Synthesizing Exposure

We just learned that the risk exposure of a call is that of the underlying multiplied by the delta. Let’s turn this idea upside-down: How many calls on Coke do you need in order to establish the same risk exposure as the cash stock position? Let’s make this more concrete. You want to own 100,000 shares of Coke, priced currently at 45/share, because you expect the share price to risk significantly before three months. You recognize, of course, that you will be exposed to the risk of Coke shares falling in price. How many options will give you approximately the same profit should Coke appreciate, and produce approximately the same loss should it depreciate?

The answer is the opposite of the above: 1 divided by the delta. If, for example, a 50 cent increase in Coke results in only a .4×50 cents=20 cents increase in the call premium, in order to produce the same 50 cents increase you need 2.5 times that. So, for 100,000 shares, you need:

100,000 / .4 = 250,000 calls

If the option is deep in-the-money, its delta is almost 1. Achieving the same risk exposure as a share of Coke, therefore, requires 1/1 = 1 call. Why? Because if the delta equals 1, then the call and the underlying are essentially the same security.

EXERCISES

1. Consider a three-month call on Coke struck at 55. With the spot price at 45, its delta is .3. What is risk exposure of 100,000 options?
2. How many of these calls are necessary to achieve same risk exposure of 100,000 shares?
3. You own 10,000 calls with 50 strike, 7,500 calls with 55 strike. What is risk of portfolio?
4. How many 50-calls are necessary to produce same risk exposure as 10,000 55-calls?

Some Additional Observations about Delta

Return to Figure 1. Notice the “smoothness” of the premium curve. As the underlying asset price increases, the slope of the curve – the option’s delta – rises continuously, from barely above 0 to barely below 1. Now look at the intrinsic value “curve.” Its slope is zero until the underlying’s spot price hits the strike, after which it jumps to 1 where it remains throughout.

This contrast between the curves highlights the following observation. The above risk exposure discussion showed that a call’s delta measure the degree to which changes in the underlying asset’s priced are mirrored by the option. The delta, therefore, may be interpreted as the call’s rate of “participation.” At .5, the call captures 50% of Coke’s price movement, up or down. As Coke rises, and the delta increases to, say, .6., 60% of Coke’s *next* is mimicked by the Call. And so on, until the slope reaches almost 1, at which point nearly 100% of spot price changes are copied. In this deep-in-the-money region, the call is essentially a duplicate of the stock.

What about the difference between 1 and the delta value – the portion of the underlying’s price movement *not* captured by the call? If the delta is .6 and the underlying experiences a price decline of, say, half a point, the call drops by only .3 (.6×.5). The call owner was not hurt by the remaining .2 decline in the spot price. One minus delta, therefore, represents the call’s degree of “protection.” As Coke declines further, the delta falls. At .25, for example, the call provides 75% protection to the owner. The protection increases (the participation decrease) as Coke continues to fall, until the slope is barely above 0 – nearly total protection, essentially no participation. In this out-of-the-money region, the option and the undelaying have, for all intents and purposes, decoupled.

Put Delta.

Figure 3 is almost the mirror image of figure 1.[[15]](#footnote-15) As Coke declines from a very high price relative to the option’s strike, say 100, the put begins to have value. Its value rises as Coke falls, and gathers steam continuously. The slope of the curve is the put’s delta. Not only does the put price rise as Coke shares fall, the slope becomes more negative. Or, the delta increases in “absolute value.” For the remainder of this discussion, we will mean absolute value when we refer to the put’s delta in descriptive terms. When quantifying the slope, we will be careful to use the minus sign. In the far out-of-the money region, the put’s delta is essentially 0. When the put is approximately at-the-money, its delta is −.5. It continues to increase as the underlying price fall, until the slope is nearly −1.

Suppose you own 50,000 put options on GE shares. GE is 27/share and the put has six months to expiration with a strike of 26. The put’s delta is −.45 (it is slightly out-of-the-money, hence the delta is just under −.5). The put’s exposure follows:

∆Put = delta×∆Stock

As the delta is negative, an increase in GE’s price causes the put to decline. For example, If GE appreciates to 28, your put position declines in value by:

−.45×1×50,000 = $22,500

A *decline* in price of 1 dollar in GE *increases* the value of the position by $22,500 (−.45×−1×50,000). Think about an actual *short* 50,000 share position in GE. An increase or decrease of 1 point in GE shares produces a $50,000 loss or gain, respectively. Thus, the put’s risk exposure equals 45% of the short’s risk exposure.

How many puts would be necessary to duplicate – synthesize – a 50,000 share short position in GE? The inverse of the above relationship, or 50,000/.45 = 111,111. By contrast, a deep in-the-money put (e.g., a strike of 60 when GE is 27) has a delta nearly −1. Approximately 50,000 puts would suffice to produce the same risk exposure of 50,000 GE shares sold short.

Writing Calls and Puts

Suppose you wrote the Coke call option analyzed earlier. Here is the relevant information:

Coke spot price: 45

Exercise price of call: 50

Expiration of call: 3 months

Premium: 1

Delta: .4

When you sold the call, you pocket 1/share. In chapter 2 we analyzed your exposure on the expiration date – the option’s payoff. What is your exposure *now*?

If Coke shares increase from 45 to 46, the call rises in value by .4×1 = .4, as explained above. You’re short the call, so you lose .4.[[16]](#footnote-16) That is, since the call premium is now 1+.4=1.4, in order for you to exit your position, you need to buy the call at the higher price, resulting in a loss of 40 cents. Conversely, if Coke declines by a dollar, the call falls by 40 cents, to 60 cents. It will cost you .6 per option to cover. As you wrote the call for 1, your profit is .4, or 40% of the decline in the spot price. So, what is your risk exposure? Equivalent to a short position in Coke, but delta × the number of written calls.

Using the same logic, you can see that writing a put presents the same risk exposure as *buying* delta shares. Consider:

Coke spot price: 45

Exercise price of put: 50

Expiration of call: 3 months

Premium: 7

Delta: −.6

Writing the put gives you 7, a substantial premium because it is in-the-money. If Coke falls by a dollar, the put *increases* in value by 60 cents. You’re short, so you need to pay 7.60 in order to cancel your position, resulting in a 0.6 loss. An increase in Coke price to 46 pushes down the put to 6.4. Your short position now shows a *profit* of 60 cents. Writing a put, you see, is equivalent to a *long* position on the underlying asset, to the tune of delta × the number of written puts.

SUMMARY BY WAY OF QUESTION

Buying a call or put presents exposure similar to owning or being short, respectively, the underlying asset to a degree measured by the option’s delta. Writing a call or put presents the reverse exposure - short and long the underlying, respectively, to the delta degree.

Coca Cola is 45 per share. Consider a call and a put, both at-the-money, each with a 3 dollar premium. As such, each delta equals .5, positive for the call, negative for the put. Buying the call gives you 50% of the exposure of buying a share of Coke – Coke appreciates by a dollar, the call goes up by 50 cents; GE depreciates by a dollar, the call falls by 50 cents. Writing the put, as we just saw, because it has the same absolute value delta but in the opposite direction presents equivalent exposure – 50% of a long position in Coke. What, then is the difference between long call and short put?

The answer to this question lies at the heart of options. While a long call and short put, with equal deltas, appear to present equivalent exposure, this is only true at the initial point. Coca Cola is at 45. It increases to 46, raising the call from 3 to 3.50. Now look back at Figure I. As the underlying asset rises in price – in particular, as it crosses the strike – the delta increases. In our case, say, from .5 to .6. The *next* increase in Coke’s price will bring forth a 60% change in the call premium. And the increase following that will produce an even higher percentage reaction from the call. Should Coke drop in price from 45 to 44 per share, the call declines from 3 to 2.50. As the Figure shows, the delta is now lower. A further decline in Coke will cause the call to decline by, say, only 40% of that amount. Now look at Figure III. Coke increases from 45 to 46. With a delta of −.5, the put falls from 3 to 2.50. Remember, you’re *short* the put, so you’ve made a 50 cent profit, 50% of the increase in Coke’s spot price. But the delta is now *lower*, say −.4. The next dollar increase in Coke share price gives you a 40 cent profit, only 40% of the increase in the underlying. And the next increase will produce an even smaller change in the put premium, hence profit for you. Should Coke decline in price instead, from 45 to 44, the put increases in value from 3 to 3.50. You lose 50 cents. As the Figure shows, the delta is now higher in absolute value. Therefore, a further dollar decline in Coke will produce a greater loss in the short put position. The call and put couldn’t be more different![[17]](#footnote-17) This dynamic – the changes in delta as the underlying asset moves in either direction – is known as “gamma.” We take this up, along with its profound implications, in chapter qqq.

TIME VALUE AND THETA

The analysis above which developed the option curve – for both calls and puts – made clear the centrality of the strike price to the value of the option. The closer the underlying’s spot price to the exercise price, the greater the chance of crossing it by the expiration. If a call option is out-the-money, the smaller the distance to the strike, the higher the probability of and greater the value of crossing it and participating in the underlying’s appreciation. If it is in-the-money, the shorter the distance to the strike, again the higher the probability of crossing it and having the benefit of protection. The same is true of a put, except the participation refers to a fall in the underlying’s price and protection is from an appreciation.

The contribution of time to an option’s value should now be clear. The longer the time to expiration, the greater the chance of the underlying crossing the strike *regardless of its current price*. An option’s value increases, therefore, the longer to expiration. Because the intrinsic value is unaffected by time, this factor is relevant only to the time value component of an option’s premium. Consider a six-month call on GE struck at 27, and a one-year call with the same parameters. GE is 25. Each call has value only to the extent that GE might cross 27 before its expiration. The first call has six months for GE to do so. The second has a full year. Clearly, the second presents a greater opportunity, as whatever occurs during the first six months obviously occurs during the year, and more. The one-year call, therefore, has more time value than the six-month. As they have the same intrinsic value (0 in this case), the one-year’s premium exceeds that of the six-month.

It is important to understand time value from another perspective. Consider the above one-year call on GE. Assume six months go by and nothing changes – neither the spot price nor its volatility. The call is now a six-month option. As just discussed, its price is lower than it was six months ago. Three months later it becomes a three month option with, all else the same, a lower premium. In short, an option declines in value as it approaches expiration. A few points surrounding this idea need to be mentioned:

1. The quantitative effect of time on an option is known as its “theta.”[[18]](#footnote-18) Theta≡△premium/△time remaining, measured in years.
2. Figure V shows the three call premium curves. As is evident, theta is greatest for options that are near-the-money. This is not surprising. As we’ve seen, time value is at its peak near-the-money. Simply stated, near-the-money options have more to lose as time progresses.
3. Time value is very low for deep-in and far-out-of-the-money options. Figure V shows that at these extreme points the differences among the three options are minimal. Theta is nearly 0.
4. The most surprising observation, at first blush, is the following. Although increasing time to expiration raises the value of calls and puts, successive increases in time have less and less effect on the premium. This is almost intuitive. Adding a week to a one-week option does a lot. Adding a week to a five-year option does next to nothing. But there is more to it than that. Options have a *maximum value*. A call cannot be worth more than the underlying spot price. A put’s premium can never be above its exercise price. Adding the call=spot price line as the upper bound to the options curves in Figure VI makes it clear that continuing to add time will ultimately do nothing for a call. And adding the put=exercise price line as the upper bound to the options curves in Figure VI does the same for a put.[[19]](#footnote-19)

VOLATILITY AND VEGA

Understanding volatility’s place in an option’s premium is now relatively straightforward. Let’s start by thinking of volatility rather simplistically: the likelihood of change and the associated degree of change. The more volatile the underlying asset’s price, the greater the chance of crossing the strike, from either direction. Calls and puts respond in the same direction to volatility. Volatility’s impact on an option’s premium, therefore, is similar to that of time to expiration. All else the same, greater volatility translates into a higher premium, lower volatility into a smaller premium. This simple, yet crucial, idea manifests itself in a number of ways, two of which we’ll consider here.

* Assume given Coke’s spot price of 44, investors have formulated a view about – we might say “estimated” – Coke’s volatility over the next, say, six months. Six month calls and puts, for all strikes, will reflect this volatility. Suppose that an event occurs, which causes investors to raise their volatility estimate. The event may be unique to Coca Cola Company or related to the stock market overall. Coke option premiums will increase. As with time to expiration, the greatest increase will be for near-the-money-options.
* Compare two options with identical parameters (exercise price, expiration, style, etc.) on different assets, for example, a utility stock and a high tech stock. Assume the two spot prices are equal. As high tech shares exhibit greater volatility, its calls and puts will command a greater premium than the utility company options.

The quantitative effect of a change in investors’ estimate of volatility on an option’s premium is “vega” (≡△premium/△volatility). As such, volatility requires a quantitative measure. It needs a number, just like the price of the underlying in delta and years to maturity in theta. We’ll develop this number in chapter qq, but in case you can’t wait, volatility is the standard deviation of the change in the asset’s price.[[20]](#footnote-20) Notice the word “change” in the definition. In light of the above discussion, this makes eminent sense, as the option has value to the extent that the underlying’s price changes.

A few important points about volatility before we close this section. We’ll expand upon all of them in later chapters. I’ll mention them briefly here. The relevant volatility measure is that expected by investors for the future, in particular, over the period until the option’s expiration. This is known, therefore, as expected volatility. The only volatility we can actually *know* is the volatility that actually occurred over some *past* time period. This is known as historical volatility, and is a statistical measure. Market participants use this historical information to form their opinion about future volatility. This future volatility, therefore, becomes “priced in” to the option’s value. If we had some pricing equation, or model, that relates an option’s price to its parameters, including volatility, then given the observed price, we can calculate the volatility that is implied by it. This is known as implied volatility.[[21]](#footnote-21)

Volatility and time to expiration enter into an option’s value interactively. Time has value only if there is volatility. And an increase in time to expiration is enhanced to the extent of volatility. Similarly, volatility provides value to an option only if there is time within which the volatility can play itself out. And the longer he time to expiry, the more an increase in volatility raises the option premium. Thus, an option’s time value should really be referred to as its time-volatility value.

After put: Effect of time on fair forward, hence on arb & time value.

Effect of time reduced somewhat by time value of money. Place this thought in theta section.

Curve apporoaches lines as approach expiry, so region of non-0,1 delta compresses. Also, as time &/or vol increase, region of delta nearly 1 expands.

The far and deep regions depend on the strike. So does the curvature.

I Consider an OTM and an ATM call on XYZ, both six-months to expiration. One week goes by and nothing else changes. Which falls more in value?

II You own a six-month NTM (near-the-money) call on DD (DuPont Corp). At the close of trading on Friday afternoon, the call is at 3. You wake up (late) Monday morning and DD is unchanged, and so is the call! What must be the explanation?

III You and a friend are both bullish on oil for the next year (because you expect a global economic recovery). But your friend is less tolerant of risk than you are. Which of you is more likely a candidate for a one-year call option than an outright purchase of the oil ETF, and why?

IV ABC is now 50/share. You believe it will decline 10% over the next few days. Explain why writing a 3-mnonth call on ABC, exercise price of 40, which you intend to close out in a week whether you’re right or wrong, is a sensible strategy.

1. Remember, we are dealing with European-style options. Hence, the reason the call is worth at least 50 is not due to the call owner’s ability to exercise now and immediately realize a profit of 100 – 50. Rather, we are making the simplifying assumption in this chapter that the arbitrage value equals the intrinsic value, in this case, 50. (Recall, this requires either ignoring dividends and the interest rate factor in an option’s premium, or that the stock’s dividend yield equals the relevant interest rate.) [↑](#footnote-ref-1)
2. By contrast, nobody would be willing to pay more than the “intrinsic value” of the stock itself. Why pay more than 100 when the market price is 100/share! [↑](#footnote-ref-2)
3. Mathematically, the second derivative of the curve is greater than 0. [↑](#footnote-ref-3)
4. “Approximately” because the fair forward price is not necessarily equal to the spot price, though in this chapter we assume equality. See footnote 1. [↑](#footnote-ref-4)
5. Mathematically, the option value curve asymptotically approaches the intrinsic value, or the slope asymptotically approaches 1. [↑](#footnote-ref-5)
6. There is actually an additional, more subtle, factor at work as time passes. Recall from chapter qq that the asset’s fair forward value is Sx[1+(r−div)]t, where S is the asset’s spot price, r the relevant interest rate (cost of carry), div the dividend yield and t is the time to maturity. Fair forward, therefore, changes over time even as (more correctly, under the assumption that) the spot price does not. In turn, this implies that the call’s arbitrage value (see chapter qq) changes as the expiration date nears. According to the assumption in footnote 1, however, this factor is absent. [↑](#footnote-ref-6)
7. Actually, the present discounted value of $500,000. This assumes no dividends are to be paid before expiration. [↑](#footnote-ref-7)
8. Notice that for both the put and the call, the argument for maximum optionality value at-the-money is nearly identical to that in chapter xx at expiration. [↑](#footnote-ref-8)
9. Sometimes market participants describe the slope in this region as 1, neglecting to say “minus.” They mean its absolute value. [↑](#footnote-ref-9)
10. It becomes more negative; e.g., from −0.3 to −0.4. [↑](#footnote-ref-10)
11. As we will see, we are implicitly assuming no decrease in Coke price volatility and little direct negative effect of the passage of time. [↑](#footnote-ref-11)
12. As shown, the time values in Table III and Table IV are equal, displaying a symmetry around 50, the exercise price of both options. This is typically not the case (though it is approximately correct). The reason, as we will see in chapter zz, is that the symmetry occurs when the fair forward – not spot – price is equal to the strike and then moves away from it in either direction.. [↑](#footnote-ref-12)
13. As mentioned, this is an approximation (but not a bad one). The call premium-spot price relationship follows a curve (it is not linear). The upward and downward reactions, therefore, are not symmetric – otherwise, how could the slope change! We’ll study this, and its implications, in the next chapter. [↑](#footnote-ref-13)
14. We will see in chapter ?? that volatility is intimately related to standard deviation. [↑](#footnote-ref-14)
15. “Almost” because whereas Coca Cola shares, at least theoretically, have no upper price bound, the most the share price can fall to is 0. The difference between the two is actually more profound than this simple observation suggest. Once the company is bankrupt – the shares are worth zero – the share price cannot rebound (the company can be “reborn,” but it is a different entity with different shares, hence options). In other words, the put option expires prior to expiration. We’ll analyze this idea in chapter ??. [↑](#footnote-ref-15)
16. The resultant .4 decline in the call’s premium is only approximate. As we learned above, the delta changes as Coke falls. The text initially assumes a constant delta for simplicity. The non-constancy of the delta is recognized later in the section. [↑](#footnote-ref-16)
17. This should not be so surprising. As we saw in chapter 2, the respective call and put payoffs – their values on the expiration date – are starkly different. Notice, however, that the difference becomes more evident as the price of the underlying moves further away from the strike price, in either direction. We really said the same thing in the text, which looks at the option values prior to expiration – the deltas become more and more different as the spot price moves away from the strike. [↑](#footnote-ref-17)
18. Theta is a positive number – more time to expiration, greater premium. As the calendar turns, time decreases, hence the premium falls. [↑](#footnote-ref-18)
19. Another way to see that adding more and more time adds less and less value to a premium is to recognize that the call and are “anchored” by their extreme points. In Figure VI each call is worth 0 when the underlying is worth 0, and the curve slope is nearly 1 when deep in-the-money. If, for argument’s sake, the call premium were to pierce though the call=spot price line, there would have to be a point where the slope exceeds 1 before falling back. But this implies that the time value – the chances of crossing back through the strike price - *increases* as the underlying moves further from the strike! In Figure VII each put is worth the exercise price when the underlying is worth 0, and the curve slope is nearly 0 when far out-of-the-money. If, for argument’s sake, the put premium were to pierce though the put=strike price line, there would have to be a point where the slope is negative, which implies a lower put value even as the underlying declines! [↑](#footnote-ref-19)
20. Well, not quite – it’s almost that, as we’ll see in chapter qq. [↑](#footnote-ref-20)
21. Note – crucially – that future volatility is not observable (only once it becomes historical!). Implied volatility tells us the market’s expectations regarding future volatility only if the option pricing equation or model is perfectly correct. More on this in chapter qq. [↑](#footnote-ref-21)